Rayat Shikshan Sanstha's

Karmaveer Bhaurao Patil Mahavidyalaya, Pandharpur

(Autonomous)

Second Year Syllabus under Autonomy

NAAC Reaccredited 'A+' grade, CGPA: 3.51

Granted under FIST-DST and The Best College

Affiliated to the

Punyashlok Ahilyadevi Holkar Solapur University, Solapur

Program: B.Sc. II

Subject: Mathematics

Semester: III and IV

Pattern: Choice Based Credit System (CBCS)

Syllabus to be implemented from June, 2020 onwards

Rayat Shikshan Sanstha's

Karmaveer Bhaurao Patil Mahavidyalaya, Pandharpur
(Autonomous)
Faculty of Science
Choice Based Credit System (CBCS)
(w.e.f. 2020-21)
Syllabus for B.Sc. Part-II
Mathematics

General Objectives of the Course:

1. To provide the students a broad understanding of the subject with emphasis on theory based practical's
2. The syllabus framing is done according to UGC norms
3. To promote understanding of basic and advanced concepts in Statistics
4. To develop curiosity, interest of the students to direct them to higher studies in the subject.
5. To assess the students by continuous internal examinations and semester examinations so as to keep the students aware throughout the year.
6. To encourage the students for study by conducting online tests, seminar, surprise test, mid-term test, open book test.
7. To monitor the quality of learning by evaluating day to day performance

1. **Duration:** The course shall be a full time.
2. **Pattern:** Semester examination.
3. **Structure of Course:**

B. Sc.-II: Semester –III				
Paper No.	**Paper Code No.**	**Title of Paper**	**Total Lectures**	**Total Credits**
Paper V	KBP-S-MAT-231	Real Analysis-I	38	2.0
Paper VI	KBP-S-MAT-232	Algebra-I	38	2.0
B. Sc.-II: Semester –IV				
Paper VII	KBP-S-MAT-241	Real Analysis-II	38	2.0
Paper VIII	KBP-S-MAT-242	Algebra-II	38	2.0
	KBP-S-MAT-P2	Annual Mathematics Practical Examination at the end of 2nd Semester	4 lectures per Batch	4.0
	KBP-S-MAT-P3	Annual Mathematics Practical Examination at the end of 2nd Semester	4 lectures per Batch	4.0
Grand Total				12.0

Paper V-KBP-S-MAT-231- Real Analysis-I

Course Objectives: The course is designed so that

1. Students will learn to understand basic statements and able to write basic proofs according to principles of quantificational logic
2. Student will understand thoroughly and precisely the concept of “limit” in its various forms
3. Student will understand thoroughly and precisely the concepts of “limit o sequence, boundd sequence,monotonic sequence andcauchy sequence in its various forms
4. Student will understand sets, equivalent sets, finite ,countable and uncountable sets ,least upper bound axioms
5. Student will learn to show whether sequence is converges or diverges

Unit 1: Sets and Functions

1.1 **Sets**

1.1.1 Operations on sets ,Cartesian product of sets ,Relation

1.2 **Functions**

1.2.1 **Definitions :** Function, Domain, Co-domain, Range, Graph of a function, Direct image and Inverse image of a subset under a function

1.2.2 **Theorem:** If $f: A \rightarrow B$ and if $X \subseteq B, Y \subseteq B$ then

$$f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$$

1.2.3 **Theorem:** If $f: A \rightarrow B$ and if $X \subseteq B, Y \subseteq B$ then

$$f^{-1}(X \cap Y) = f^{-1}(X) \cap f^{-1}(Y)$$

1.2.4 **Theorem:** If $f: A \rightarrow B$and $X \subseteq A$, $Y \subseteq A$ then $f(X \cup Y) = f(X) \cup f(Y)$

1.2.5 **Theorem :** If $f: A \rightarrow B$ and $X \subseteq A$, $Y \subseteq A$ then $f(X \cap Y) \subset f(X) \cap f(Y)$

1.2.6 **Definitions :** Injective, Surjective function and , Bijective function (1-1 correspondence), Inverse function.

1.2.7 **Theorem :** Composition of two bijective functions is a bijective function.

1.3 **Countable Sets**

1.3.1 **Definitions:** Finite sets, Infinite sets, Countable Sets, Uncountable Sets.

1.3.2 **Examples of Countable sets :** Set of Natural numbers, Set of Integers, Cartesian product of Countable sets

1.3.3 **Theorem :** Countable union of countable set is countable

1.3.4 **Theorem :** Set of Rational numbers is countable

1.3.5 **Theorem :** Any subset of countable set is countable

1.3.6 **Theorem :** The closed interval [0,1] is uncountable

1.3.7 **Theorem :** The set of all real numbers is uncountable

1.3.8 **Theorem :** Every infinite set has a countably infinite subsets

1.3.9 **Examples**

Unit 2: Completeness Property of $\mathbb{R}$ **[08]**

2.1 **Definitions :** Lower bound ,Upper bound of a subset of $\mathbb{R}$, Bounded set, Supremum $(l.u.b)$, Infimum $(g.l.b)$

2.2 **Least Upper Bound Axiom [Completeness Property of $\mathbb{R}$]**

2.3 **Theorem (Archimedean Property) :** If $x \in \mathbb{R}$ then there exists $n_x \in \mathbb{N}$ such that

$$x \le n_x$$

2.3.1 **Corollary :** If $S = \{\frac{1}{n} : n \in \mathbb{N}\}$ then inf $S = 0$

2.3.2 **Corollary :** If $t > 0$ then there exist $n_t \in \mathbb{N}$ such that $0 < \frac{1}{n_t} < t$

2.3.3 **Corollary :** If $y > 0$ then there exist $n_y \in \mathbb{N}$ such that $n_y - 1 < y < n_y$

2.4 **Theorem :** There exists a positive real number x such that $x^2 = 2$

2.4.1 **Corollary :** If x and y are real numbers with $x < y$ then there exist an irrational number z such that $x < z < y$

2.5 **Intervals**

2.5.1 **Characterization Theorem :** If S is a subset of $\mathbb{R}$ that contains at least two points and has the property
If $x, y \in S$ and $x < y$ then the closed interval $[x, y] \subseteq S$ where S is an interval

Unit 3: Sequence of Real Numbers [14]

3.1 **Sequence and Subsequence**

3.1.1 Definition : Sequence , Subsequence and examples.

3.2 **Limit of a Sequence**

3.2.1 **Definition**

3.2.2 **Theorem :** If $\{S_n\}_{n=1}^{\infty}$ is a sequence of non-negative numbers and if $\lim_{n\to\infty} S_n = L$ then $L \ge 0$

3.3 **Convergent Sequence**

3.3.1 Definition : Convergent and Divergent Sequence.

3.3.2 **Theorem :** Convergent sequence cannot converge to two distinct points

3.3.3 **Theorem (Without Proof) :** If sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent to L then any subsequence of $\{S_n\}_{n=1}^{\infty}$ is also convergent to L

3.4 **Operations on Convergent sequences**

3.4.1 **Theorem :** If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers,
if $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} t_n = M$ then $\lim_{n\to\infty}(s_n + t_n) = L + M$

3.4.2 **Theorem :** If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers,
if $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} t_n = M$ then $\lim_{n\to\infty}(s_n - t_n) = L - M$

3.4.3 **Theorem :** If $\{s_n\}_{n=1}^{\infty}$ is a sequences of real numbers,
If $c \in \mathbb{R}$ and if $\lim_{n\to\infty} s_n = L$ then $\lim_{n\to\infty} cs_n = cL$

3.4.4 **Theorem :** If $0 < x < 1$ then the sequence $\{x^n\}$ converges to 0

3.4.5 **Lemma :** If sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent to L then $\{{s_n}^2\}_{n=1}^{\infty}$ converges to L^2

3.4.6 **Theorem :** If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers,
if $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} t_n = M$ then $\lim_{n\to\infty}(s_n . t_n) = LM$

3.4.7 **Theorem :** If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers,
if $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} t_n = M$, M $\neq 0$ then
$\lim_{n\to\infty}(s_n / t_n) = L/M$, provided M $\neq 0$
If $\{t_n\}$ is bounded sequence.

Unit 4: Monotone Sequences and Cauchy Sequences **[09]**

4.1 **Monotone Sequence**

4.1.1 **Definition and Examples**

4.1.2 **Theorem :** A non-decreasing sequence which is bounded above is convergent

4.1.3 **Theorem :** A non-increasing sequence which is bounded below is convergent

4.1.4 **Corollary :** The sequence $\{(1+\frac{1}{n})^n\}$ is convergent

4.1.5 **Theorem (Without Proof) :** A non-decreasing sequence which is not bounded above diverges to infinity

4.1.6 **Theorem (Without Proof) :** A non-increasing sequence which is not bounded below diverges to infinity

4.1.7 Definition : limit superior and limit inferior and Theorem(without proof)

4.1.8 **Theorem :** A bounded sequence of real numbers has convergent subsequence

4.2 **Cauchy Sequence**

4.2.1 **Definition and Examples**

4.2.2 **Theorem :** If sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ converges then $\{s_n\}_{n=1}^{\infty}$ is Cauchy sequence

4.2.3 **Theorem :** If $\{s_n\}_{n=1}^{\infty}$is the Cauchy sequence of real numbers then $\{s_n\}_{n=1}^{\infty}$is bounded

4.2.4 **Theorem :** If $\{s_n\}_{n=1}^{\infty}$is the Cauchy sequence of real numbers then $\{s_n\}_{n=1}^{\infty}$is Convergent

Learning Outcomes : Student will have

1. An ability to work within an axiomatic framework

2. A detailed understanding of how monotone sequence converges and completeness property of $\mathbb{R}$ and ability to explain the steps in standard Mathematical notations.

3. Knowledge of some simple techniques for testing the convergence of sequences confidence in applying them;

4. Familiarity with a variety of well-known sequences with a developing intuition about the behavior of new ones;

5. Be able to understand logical arguments and logical constructs. Have a better understanding of sets, functions and relations

Recommended Books:

1. **R.R.Goldberg**,Methods of real Analysis, Oxford & IBH Publishing co. Pvt. Ltd, New Delhi(UNIT 1,2,3,4)

2. **S.C.Malik and SavitaArora**, Mathematical Analysis(Fifth Edition), New Age International (P) Limited, 2017(UNIT 1,2,3,4)

3. **T. M. Apostol**,Calculus (Vol.I) ,John Wiley and sons (Asia) P.Ltd.2002

Reference Books :

1. **R.G.Bartle and D.R.Sherbert**, Introduction to Real Analysis, Wiley India Pvt. Ltd, Fourth Edition, 2016
2. **D.Somasundaram and B Choudhary**, First Course in Mathematical Analysis, Narosa publishing house New, Delhi, Eighth Reprint 2013.
3. P.K.Jain and S.K.Kaushik, An Introduction to Real Analysis, S.Chand&CompanyLtd. New Delhi, First Edition 2000
4. Shanti Narayan and M.D.Raisinghania, Elements of Real Analysis, S.Chand& Company Ltd. New Delhi, Fifteenth Revised Edition 2014

Paper -VI-KBP-S-MAT-232 Algebra-I

Course Objectives: The course is designed so that

1. Students should understand types of Matrices and their applications
2. Students should develop the skills find the Eigen values and Eigen vectors
3. Present the divisibility and relationship between the Greatest common divisor and Least common multiple
4. Define Types of Matrices, Divisibility in Integers, Equivalence relation and partitions and Congruence relation
5. Present the concept Group and its basic properties

Unit 1: Matrices **[12]**

1.1 **Introduction**

1.1.1 Definition with Illustration.

1.1.2 Types of Matrices

1.1.3 Definitions: Transpose of Matrix, conjugate of Matrix, Symmetric Matrix , Asymmetric Matrix

1.2 **Hermitian and Skew hermitian**

1.2.1 **Definitions :**Hermitian and Skew Hermitian

1.2.3 **Theorem:** The necessary and sufficient condition for a matrix A to be Hermitian is that$A = A^{\theta}$.

1.2.4 **Theorem:** The necessary and sufficient condition for a matrix A to be Skew Hermitian is that $A^{\theta} = -A$.

1.2.5 **Theorem :** If A and B are Hermitian (Skew Hermitian) then A+B is also Hermitian (SkewHermitian).

1.2.6 **Theorem :** If A is Hermitian then iA is Skew Hermitian.

1.2.7 **Theorem :** If A is Skew Hermitian then iA is Hermition.

1.2.8 **Theorem :** Every square Matrix is uniquely expressed as the sum of Hermitian Skew hermitian matrix.

1.3 **Eigenvalues and Eigenvectors**

1.3.1 **Definitions :** Minor, Cofactor, Adjoint of matrix, Rank of a matrix, Inverse of a matrix,Characteristics Polynomial of matrix.

1.3.2 **Eigenvalues and Eigenvectors**

1.4 **System of a linear Equations**

1.4.1 System of Homogeneous linear Equations.

1.4.2 Nature of solutions of $AX = 0$.

1.4.3 Examples on 1.4.1.

1.4.4 System of Non -homogeneous linear Equations.

1.4.5 Nature of Solution of $AX = B$.

1.4.6 Examples on 1.4.4.

1.5 **Cayley-Hamilton Theorem (Statement only)**

1.5.1 Finding inverse of matrix using Cayley-Hamilton Theorem.

Unit 2 Divisibility in Integers **[10]**

2.1 **Definition :Divisibility in integers**

2.2 **The well ordering principle(Statement Only)**

2.3 **Properties of Divisibility**

2.3.1 Definition of divisor and Multiple

2.3.2 **Theorem:** Let a,b,c,d be integers. Then

i) If a | b then a | bx

ii) If a | b and a | c then a | bx+cy $\forall$ x, y $\in$ I

iii) If a | b and b | c then a | c

iv) If m $\neq$ 0 is in $\mathbb{Z}$ and a | b $\Rightarrow$ am | bm

v) If a | b and c | d then ac | bd

vi) If ab | bc then a | c ; (b$\neq$ 0)

2.4 **Theorem: Division Algorithm (Without Proof)**

2.5 **Greatest common divisor and least common multiple**

2.5.1 **Definitions:**Greatest common divisor and least common multiple

2.5.2 **Theorem :** Let a and b be two integers at least one of them not 0. Then there exist A unique greatest common divisor d of a and b. Moreover, d can be written as $d = am + bn$ for integers m and n.

2.5.3 **The Euclidean Algorithm and Examples.**

2.5.4 Definition : Relatively Prime

2.5.5 **Euclid's lemma :** For a prime number p , if p | ab then either p | a or p | b.

2.6 **Theorem : (Unique Factorization Theorem or Fundamentals Theorem of Arithmetic) (Without Proof and examples only)**

Unit 3 Relation **[12]**

3.1 **Relation**

3.1.1 **Definitions:** Cartesian Product, Relation, Binary Relation, Inverse Relation.

3.1.2 Examples on 3.1.1.

3.2 **Pictorial Representation of Relation**

3.2.1 Co-ordinate Diagram

3.2.2 Arrow Diagram

3.2.3 Matrix Representation

3.2.4 Directed Graph

3.2.5 Examples on 3.2.1 to 3.2.4

3.3 **Composition of Relations**

3.3.1 **Definition:** Composition of Relations

3.3.2 **Theorem:** Let A, B, C and D be sets. Let R: A$\longrightarrow$ B , S: B$\longrightarrow$ C , T: C$\longrightarrow$ D

be

Relation then $R \circ (S \circ T) = (R \circ S) \circ T$.

3.4 **Types Of Relations**

3.4.1 **Definitions :** Reflexive, Symmetric, Antisymmetric and transitive.

3.4.2 Examples on 3.4.1

3.4.3 **Theorem:** Let R be relation on set A. Then R^{∞} is the smallest transitive

relation

3.4.4 on A that contain R.

Definition : Transitive closure and examples.

3.5 **Equivalence relation and partition**

3.5.1 **Definition :** Equivalence relation and partition

3.5.2 **Theorem:** Let R be an equivalence relation on setA. Then quotient set A / R forms a partition of A .

3.5.3 **Theorem:** Let $\{A_i\}, i \in I$ be partition of a set A. Then there exists an equivalence Relation Ron the set A such that quotient set A / R is the given partition $\{A_i\}, i \in I$ on A.

3.6 **Partial order relation**.

3.6.1 **Definition**: Partial order relation.

3.6.2 Examples on 3.6.1

3.7 **Congruence relation on Integers**

3.7.1 **Definition** : Congruence relation

3.7.2 **Congruence arithmetic**

3.7.3 **Theorem:** Let $n > 1$ be a fixed positive integer and a, b, c, d be arbitrary integers then the following conditions holds

$i)$If $a \equiv b(\text{mod} n)$ and $c \equiv d(\text{mod} n)$then $a + c \equiv b + d(\text{mod} n)$.

$ii)$If $a \equiv b(\text{mod} n)$ and $c \equiv d(\text{mod} n)$ then $ac \equiv bd(\text{mod} n)$.

3.7.4 **Examples**

Unit 4: Groups **[11]**

4.1 **Binary operation on a set**

4.1.1 **Definition** :Binary operation on a set with illustration

4.2 **Semigroup**

4.2.1 **Definition :**Semigroup with illustration

4.3 **Monoid**

4.3.1 **Definition :**Monoid with illustration

4.4 **Group**

4.4.1 **Definition:** Group, Finite Group, Infinite Group, Order of Group, Abelian Group.

4.4.2 Examples on 4.4.1

4.5 **Properties of Groups**

4.5.1 **Theorem :** If $< G,* >$is a group, then

a) Identity element in G is unique

b)Every a in G has unique inverse in G.

c) For every a in G, $(a^{-1)})^{-1} = a$

d) For all a, b ∈ G , $(a * b)^{-1} = b^{-1} * a^{-1}$

4.5.2 **Theorem :**If a, b, c are elements in a group G, then

i) $a * b = a * c$ implies $b = c$ (Left Cancellation Law)

ii)$b * a = c * a$ implies $b = c$ (Right Cancellation Law)

4.5.3 **Theorem :** If G is a group and a,b∈ G, then the equations $a * x = b$ and $y * a = b$ have unique solutions $x = a^{-1} * b$ and $y = b^{-1} * a$ respectively.

4.5.4 **Definition :** Order of element with illustration, Properties(Without Proof)

4.6 **Permutations**

4.6.1 **Definition with Illustration**

4.6.2 **Cyclic Permutation**

4.6.3 **Transposition , Disjoint Permutations , Even and Odd Permutations**

Learning Outcomes :

1. After successful completion of Matrices students will solve the system of equations using the language of Matrices
2. Understand the elementary concepts of Matrices,System of a linear Equations,Greatest common divisor and least common multiple, Partial order relation and basic structure of Group
3. Elementary number theory is the study of the basic structure and properties of integers
4. Learning Matrices and Divisibility of integers helps improving one's ability of Mathematical Thinking

Recommended Books :

1. **Shantinarayan**, A Text Book of Matrices ,S. chand Co.,Pvt. Ltd. Raminagar, New Delhi. (Unit 1)
2. **David. M. Burton,** Elementarary Number Theory ,7th Edition.2017 McGraw Hill Education (Unit 2)
2. **Schaum'sOutline**, Discrete Mathematics,(3rd Edition) : Seymour hipschutz,Marehipson , Tata MaGraw-Hill Publishing Company Ltd., New Delhi.(Unit)
3. **V.K. Khanna and S. K. Bhambri** , A course in abstract Algebra , Vikas Publishing house Private Limited ,New Delhi , Fifth Edition 2016 (Unit 4)

Reference Books :

1. **J.B.Fraleigh**, A first couse in abstract Algebra ,Narosa Publishing House New Delhi, Tenth Reprint 2003
2. **A.R. Vasishtha** ,Modern Algebra ,Krishna Prakashan ,Meerut 1994
3. **M.Artin** ,Algebra , Prentice hall of India ,New delhi,1994
4. **I.N. Herstein** ,Topics in Algebra ,Wiley India Pvt. Ltd.

Paper -VII-KBP-S-MAT-241 Real Analysis-II

Course Objectives: The course is designed so that

1. Ability to work within an axiomatic framework
2. Students will be exposed to the basic ideas of Real Analysis and it is required for their subsequent course work
3. Define Limit Superior and Inferior of Sequences and tests for convergence of series
4. Sequence and series of functions are especially useful in obtaining approximations to a given function and defining new functions from known ones
5. In this paper , we shall consider sequences whose terms are functions rather than real numbers and pay attention to the general properties that are associated with the uniform convergence of sequence and series of functions

Unit 1: **Limit Superior and Inferior of Sequences** **[14]**

1.1 **Definitions and Examples**

1.1.1 **Theorem :** If $\{S_n\}_{n=1}^{\infty}$ is convergent sequence of real numbers then

$$\lim_{n\to\infty} sup\, s_n = \lim_{n\to\infty} S_n$$

1.1.2 **Theorem :** If $\{S_n\}_{n=1}^{\infty}$ is convergent sequence of real numbers then

$$\lim_{n\to\infty} inf\, S_n = \lim_{n\to\infty} S_n$$

1.1.3 **Theorem :** If $\{S_n\}_{n=1}^{\infty}$ is convergent sequence of real numbers and if

$\lim_{n\to\infty} sup\, S_n = \lim_{n\to\infty} inf\, S_n = L$ Where $L \in \mathbb{R}$ then $\{S_n\}_{n=1}^{\infty}$ is convergent and $\lim_{n\to\infty} S_n = L$

1.1.4 **Theorem :** If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are bounded sequences of real numbers and if $s_n \le t_n$ then

$i)\ \lim_{n\to\infty} sup\, s_n \le \lim_{n\to\infty} sup\, t_n$

$$ii)\ \lim_{n\to\infty} inf\, s_n \le \lim_{n\to\infty} inf\, t_n$$

1.1.5 **Theorem :** If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are bounded sequences of real numbers then

$$i)\ \lim_{n\to\infty} sup\,(s_n + t_n) \le \lim_{n\to\infty} sup\, s_n + \lim_{n\to\infty} sup\, t_n$$

$$ii)\ \lim_{n\to\infty} inf\,(s_n + t_n) \ge \lim_{n\to\infty} inf\, s_n + \lim_{n\to\infty} inf\, t_n$$

Unit 2: **Series of Real Numbers** **[09]**

2.1 **Definition and examples**

2.2 **Convergent and Divergent Series**

2.2.1 **Definitions :** Convergent Series, Divergent Series and Examples

2.2.2 If $\sum_{n=1}^{\infty} a_n$ is convergent series then $\lim_{n\to\infty} a_n = 0$

2.3 **Cauchy's General Principal for convergence(Statement only)**

A necessary and sufficient condition for the convergence of an infinite series $\sum_{n=1}^{\infty} u_n$ is that the sequence of its partial sum $\{s_n\}$ is convergent

2.4 **Series with non-negative terms**

2.4.1 **Definition and Examples**

2.4.2 **Theorem :** A non-negative term series converges if and only if its sequence of partial sum is bounded above.

2.5 **Tests for convergence**

2.5.1 **Comparison Test (First Type)**

If $\sum u_n$ and $\sum v_n$ are two series of non-negative terms and $k \neq 0$,a fixed positive real

number (independent of n) and there exists a positive integer m such that

$u_n \leq kv_n$ for every $n \geq m$ then

$a)$ $\sum u_n$ is convergent if $\sum v_n$ is convergent and

$b)$ $\sum v_n$ is divergent if $\sum u_n$ is divergent.

2.5.2 **Comparison Test (Second Type)**

If $\sum u_n$ and $\sum v_n$ are two series of non-negative terms and there exist positive real number m , such that

$(u_n/u_{n+1}) \geq (v_n/v_{n+1})$ for every $n \geq m$ then

$a)$ $\sum u_n$ is convergent if $\sum v_n$ is convergent and

$b)$ $\sum v_n$ is divergent if $\sum u_n$ is divergent.

2.5.3 **Root Test :** Consider the series $\sum_{n=1}^{\infty} a_n$ .Then

$a)$ If $\lim sup|a_n|^{1/n} < 1$ then the series converges absolutely

$b)$ If $\lim sup|a_n|^{1/n} > 1$ then the series diverges

$c)$ If $\lim sup|a_n|^{1/n} = 1$,this test gives no information

2.6 **Alternating Series**

2.6.1 **Definition**

2.6.2 **Leibnitz Test :** If the alternating series

$u_1 - u_2 + u_3 - u_4 + \ ...$, $(u_n > 0$ for every $n)$ is such that

$i)$ $u_{n+1} \leq u_n$, for every n and

$ii)$ $\lim u_n = 0$ then the series converges

2.7 **Absolute and Conditional Convergence**

2.7.1 Definitions and Examples

2.7.2 Theorem: Every Absolutely convergent series is convergent

2.8 **Examples**

Unit 3: **Sequence and Series of Functions** **[14]**

3.1 **Pointwise convergence of sequence of functions**

3.1.1 Definition and Examples

3.2 **Uniform convergence of sequence of functions**

3.2.1 Definition and Examples

3.3 **Uniform Convergence and Continuity**

3.3.1 **Theorem:** Assume $f_n \to f$ uniformly on an interval S.If each function f_n is continuous at a point P in S then the limit function f is also continuous at P.

3.3.2 **Theorem :** If series of functions $\sum u_k$ converges uniformly to a function f on a set S and if each term u_k is continuous at a point P in S then f is also continuous at P.

Unit 4: **Differentiability and Integrability of Series of Functions** **[08]**

4.1 **Theorem :** Assume that $f_n \to f$ uniformly on an interval $[a,b]$ and assume that each function f_n is continuous on $[a,b]$.Define a new sequence $\{g_i\}$ by the equation $g_n(x) = \int_a^x f_n(t)\ dt$ if $x \in [a,b]$ and $g(x) = \int_a^x f(t)\,dt$ Then $g_n \to n$

uniformly on $[a, b]$ .In symbols ,we have

$$\lim_{n\to\infty}\int_a^x f_n(t)\,dt = \int_a^x \lim_{n\to\infty} f_n(t)\,dt$$

4.2 **Theorem :** Assume that series of functions $\sum u_k$ converges uniformly to a function f on an interval $[a, b]$ where each u_k is continuous on $[a, b]$.
For $x \in [a, b]$
Define $g_n(x) = \sum_{k=1}^{n} \int_a^x u_k(t)\ dt$ and $g(x) = \int_a^x f(t)\ dt$
Then $g_n \to g$ uniformly on $[a, b]$

4.3 **Sufficient Condition for Uniform Convergence**

4.3.1 **Theorem (Weierstrass M-Test) :** Given series of functions $\sum u_k$ which converges pointwise to a function f on a set S.If there is a convergent series of positive constants $\sum M_n$ such that $0 \le |u_n(x)| \le M_n$ for every $n \ge 1$ and every x in S. Then $\sum u_k$ converges uniformly on S.

4.4 **Power Series**

4.4.1 **Definition**

4.4.2 **Interval of Convergence and its examples**

Learning Outcomes : Student will have :

1. An ability to work within an axiomatic framework
2. A detailed understanding of how Cauchy's Criterion for the convergence of sequence and series follow from the completeness property of ℝ and ability to explain the steps in standard Mathematical notations.
3. Knowledge of some simple techniques for testing the convergence of sequences and series and confidence in applying them;
4. Familiarity with a variety of well-known sequences and series, with a developing intuition about the behavior of new ones;
5. An understanding of how elementary functions can be defined by power series, with an ability to deduce some of their easiest

Recommended Books :

1. **R.R.Goldberg**,Methods of real Analysis, Oxford & IBH Publishing co. Pvt. Ltd, New Delhi (UNIT 1,2,3,4)
2. **S.C.Malik and SavitaArora**, Mathematical Analysis(Fifth Edition), New Age International (P) Limited, 2017(UNIT 1,2,3,4)
3. **Tom M Apostol**,Calculus (Vol.I) ,John Wiley and sons (Asia) P.Ltd.2002

Reference Books :

1. **R.G.Bartle and D.R.Sherbert**, Introduction to Real Analysis, Wiley India Pvt. Ltd, Fourth Edition, 2016
2. **D.Somasundaram and B Choudhary**, First Course in Mathematical Analysis, Narosa publishing house New, Delhi, and Eighth Reprint 2013.
3. **P.K.Jain and S.K.Kaushik**, An Introduction to Real Analysis, S.Chand&Company Ltd. New Delhi, First Edition 2000
4. **Shanti Narayan and M. D. Raisinghania**, Elements of Real Analysis, S.Chand&CompanyLtd.New Delhi, Fifteenth Revised Edition 2014
5. **Shanti Narayan and P. K. Mittal**, A course of Mathematical Analysis, S.Chand&Company Ltd. New Delhi, Reprint 2016

Paper -VIII-KBP-S-MAT-242 Algebra-II

Course Objectives: The course is designed so that

1) Students should understand types of subgroups and how to identify them
2) Students should develop the skills to use various groups and to prove various results
3) Present the relationship between abstract algebraic structures with familiar group theory
4) Define Subgroups, Normal subgroup, Cyclic Subgroups, Homomorphism and Permutation Group
5) Present the concept of Kernel of Homomorphism and Permutation Group structure

Unit1 Subgroups **[12]**

1.1 **Subgroups**

1.1.1 **Definition :** Subgroups with illustrations

1.2

1.2.1 **Theorem :** A non empty subset H of a group G is a subgroup of G If and only if

(i) $a, b \in H \Rightarrow ab \in H$

(ii) $a \in H \Rightarrow a^{-1} \in H$

1.2.2 **Theorem :** A non empty subset of a group G is a subgroup of G iff

$$a, b \in H \Rightarrow ab^{-1} \in H$$

1.2.3 **Theorem :** A non empty finite subset H of a group G is a subgroup of G iff H is

Closed under multiplication.

1.3 **Centre of a Group**

1.3.1 **Definition:**Centre of a Group ,Normalizer of a element with illustration.

1.3.2 **Theorem :** Centre of group G is subgroup of group G.

1.3.3 **Theorem :** Normalizer of an element group G is subgroup of group G.

1.4 **Cosets**

1.4.1 **Definition:**Coset and examples

1.4.2 **Theorem :** Let H be a subgroup of G then

i) $Ha = H \iff a \in H \text{ and } aH = H \iff a \in H$

ii) $Ha = Hb \iff ab^{-1} \in H$ and $aH = bH \iff a^{-1}b \in H$

iii) $Ha(aH)$ is a subgroup of G iff $a \in H$

1.4.3 **Theorem :**$Ha = \{x \in G \mid x \equiv a \bmod H\} = cl(a)$ for any a in G

1.5 **Lagrange's Theorem**

1.5.1 Theorem: If G is a finite group and H is a subgroup of G then $o(H)$ divides $o(G)$.

1.6 **Index of a subgroup**

1.6.1 **Definition :** Index of subgroup H in G with illustration

1.7 **Theorem :**For subgroups H and K of G,HK is a subgroup of G iff HK=KH

Unit 2 Cyclic groups **[11]**

2.1 **Cyclic groups**

2.1.1 **Definition :** Cyclic group , generator of a cyclic group

2.1.2 Examples on 2.1.1

2.2

2.2.1 **Theorem:** Order of a cyclic group is equal to the order of its generator.

2.2.2 **Theorem:**A subgroup of cyclic group is cyclic.

2.2.3 **Theorem :** Every cyclic group is abelian.

2.2.4 **Theorem :** If G is finite group then order of any element of G divides order of G.

2.2.5 **Theorem :** An infinite cyclic group has precisely two generators.

2.3 **Euler ϕ function**

2.3.1 **Definition :** Euler's $\emptyset$ function

2.3.2 **Theorem :** Number of generators of a finite cyclic group of order n is $\emptyset(n)$.

2.4 **Euler and Fermat's Theorem**

2.4.1 **Euler's Theorem :** Let $a, n (n \geq 1)$ be any integers such that $\gcd(a, n) = 1$ then $a^{\emptyset(n)} \equiv 1 (mod n)$

2.4.2 **Fermat's Theorem :** For any integer a and prime p $a^p \equiv a (mod p)$

2.4.3 Examples on 2.4.1 and 2.4.3

Unit 3 Normal groups **[11]**

3.1 **Normal groups**

3.1.1 Definitions : Normal subgroups , Simple group

3.1.2 Examples

3.2 **Results on Normal Groups**

3.2.1 **Theorem :** A subgroup H of group G is normal in G iff $g^{-1}Hg = H, g \in G$.

3.2.2 **Theorem :**A subgroup H of group G is normal in G iff $g^{-1}hg \in H$ for all $h \in H, g \in G$.

3.2.3 **Theorem :** A subgroup H of group G is normal in G iff the product of two right (left) cosets of H in G is again a right (left) coset of H in G.

3.3 **Quotient groups**

3.3.1 **Definition : Quotient groups with illustration.**

3.3.2 **Theorem :** If G is finite group and N is normal subgroup of G then $o\left(\frac{G}{N}\right) = \frac{o(G)}{o(N)}$.

3.3.2 **Theorem :** Every quotient group of cyclic group is cyclic.

Unit 4 Homomorphism , Permutation Group **[11]**

4.1 **Homomorphism**

4.1.1 Definitions: Homomorphism ,Epimorphism , Monomorphism , Endomorphism and Automorphism.

4.1.2 Examples on 4.4.1

4.1.3 **Theorem :** If $f: G \to G'$ is homomorphism then

(i) $f(e) = e'$

(ii) $f(x^{-1}) = [f(x)]^{-1}$

(iii) $f(x^n) = [f(x)]^n$, nis an integer.

4.2 **Kernel of Homomorphism**

4.2.1 **Definition:** Kernel of Homomorphism with illustration

4.2.2 **Theorem :** If $f: G \to G'$ is homomorphism then kerf is a normal subgroup of G.

4.2.3 **Theorem :** A homomorphism $f: G \to G'$ is one-one if and only if $\ker f = \{e\}$.

4.3 **Isomorphism Theorems**

4.3.1 **Fundamental Theorem of group Homomorphism :** If $f: G \to G'$ is an onto homomorphism with $K = \ker f$ then $\frac{G}{K} \cong G'$.

4.3.2 **Second theorem of Isomorphism :(Statement only)**Let H and K be two subgroups of group G , where H is normal in G, then $\frac{HK}{H} \cong \frac{K}{H \cap K}$.

4.3.3 **Third theorem of Isomorphism (Statement only):** Let H and K be two normal subgroups of group G ,such that $H \subseteq K$ then $\frac{G}{K} \cong \frac{G/H}{K/H}$.

4.4 **Permutation Group**

4.4.1 **Cayley Theorem:** Every group G is isomorphic to a permutation group.

4.4.2 **Theorem:**Set of even permutations is a normal subgroup of S_n Alternating group.

Learning Outcomes :

1. Understand the basic concepts of Subgroups and their applications in both algebraic and geometric structures
2. Explain the fundamental concepts of algebra and their role in modern algebra
3. Explain Demonstrate accurate and efficient use of advanced algebraic techniques
4. Apply problem-solving using advanced algebraic techniques applied to diverse situations in Mathematics

Recommended Books :

1. **V.K. Khanna and S. K. Bhambri** , A course in abstract Algebra , Vikas Publishing house Private Limited ,New Delhi , 3rd Edition 2008 (Unit 1,2,3,4)

Reference Books :

1. **J.B.Fraleigh**, A first course in Abstract Algebra, Narosa Publishing House New Delhi, Tenth Reprint 2003.
2. **A.R. Vasishtha** ,Modern Algebra ,Krishna Prakashan ,Meerut 1994
3. **M.Artin** ,Algebra , Prentice hall of India ,New delhi,1994
4. **I.N. Herstein** ,Topics in Algebra ,Wiley India Pvt. Ltd.

MATHEMATICS PRACTICAL-II

Learning Objectives:

After Completing this course students will be able to :

1. Understand how matrices and determinants are used as mathematical tools in QA.
2. Finding power of Matrix and determining the inverse of matrix.
3. Use Matrices to represent a system of equations.
4. Provide exposure to problem solving through programming.
5. Train the students to the basic concepts of the c-programming language.

Group A

Sr. No.	Name of the practical	No. of Practicals
1	Solution of System of m linear homogeneous equations in n unknowns	1
2	Solution of System of m linear non homogeneous equations in n unknown	1
3	Inverse of Matrix by Cayley Hamilton Method	1
4	Euclidean Algorithm	1
5	Pictorial Representation of Relation	1
6	Examples on equivalence relation	1
7	Examples on Fermat's theorem	1
8	Examples on Group & Order of an element	1
9	Beta function	1
10	Gamma function	1
Group B		
11	C-Introduction-I	1
12	C-Introduction-II	1
13	Complete Structure of C-programe	1
14	Simple C-programmes	1
15	If Statement, If else Statement & Switch Statement	1
16	While loop & do while loop	1
17	For loop	1
18	Go to, break continue statement	1
19	One Dimensional Array	1
20	Two Dimensional Array	1

Learning outcomes:

1. Critically analyze and construct Mathematical arguments that relate to the study of introductory Matrix theory.
2. Students will develop logics which will help them to create programs.
3. Students will able to develop the applications of Matrix theory.
4. Recognize the types of group when described using a standard forms.

Reference Books:

1. **Shantinarayan**,A Text Book of Matrices ,S. chand Co.,Pvt. Ltd. Raminagar, New Delhi

2. **V.K. Khanna and S. K. Bhambri** , A course in abstract Algebra , Vikas Publishing house
Private Limited ,New Delhi , Fifth Edition 2016
3.**R.B. Kulkarni, U.H. Naik, J.D. Yadhav, S.P. Thorat, A.A. Basade, H.V. Patil, H.T. Dinde**,
A Hand Book of Computational Mathematics Laboratory, Shivaji University
Mathematics Society,2005
4. **Shanti Narayan, P.K. Mittal** :Integral Calculus , S. Chand and comp. New Delhi
5.**B.P. Demidovich& I. A. Maron** Computational Mathematics,translated by George Yankosky
Mir Publishers, Moscow
6. **J.B.Fraleigh**, A first couse in abstract Algebra ,Narosa Publishing House New Delhi, Tenth
Reprint 2003
7. **A.R. Vasishtha** ,Modern Algebra ,Krishna Prakashan ,Meerut 1994
8.**Seymour hipschutz,Marehipson**Schaum'sOutline ,Discrete Mathematics,(3rd Edition), Tata
MaGraw-Hill Publishing Company Ltd., New Delhi.

MATHEMATICS PRACTICAL-III

Learning Objectives:

After Completing this course students will be able to :

1. Students should develop the skills to use various groups and to prove various results.
2. Define double integration over rectangle.
3. Sequence and series of functions are especially useful in obtaining approximations to a given function and defining new functions from known ones.
4. Change variables in multiple integrals.
5. To transform the system of equations to a new having upper triangular form which back substitution scheme.

Group A

Sr. No.	Name of the practical	No. of Practicals
1	Examples on Cyclic Group	1
2	Examples on Normal Subgroup	1
3	Permutation Group	1
4	Homomorphism and Group	1
5	Comparison test and Cauchy’s Root test	1
6	D’Almberts Ratio test and P-test	1
7	Double Integration over the given region	1
8	Double Integration : Change of order of integration	1
9	Double Integration : Change of co-ordinate axis	1
10	Double Integration by using Polar Co-ordinates	1
Group B		
11	Function	1
12	Trapazoidal Rule and its Program	1
13	Simpsons(1/3)rd rule and program	1
14	Simpsons(3/8)th rule and program	1
15	Gauss Elimination Method	1
16	Gauss Jordan Method	1
17	Gauss-Seidel Method	1
18	Euler's Method	1
19	Euler's Modified Method	1
20	Runge-Kutta second & fourth order Method	1

Learning outcomes:

1. Solve the problems involving various methods of eliminations.
2. Solve the system of linear equations by using numerical methods.
3. Students will able to develop the applications of Matrix theory.

4. Explain Demonstrate accurate and efficient use of advanced algebraic techniques.

Reference Book:

1. **Shantinarayan**,A Text Book of Matrices ,S. chand Co.,Pvt. Ltd. Raminagar, New Delhi
2. **V.K. Khanna and S. K. Bhambri** , A course in abstract Algebra , Vikas Publishing

house
Private Limited ,New Delhi , Fifth Edition 2016

3.**R.B. Kulkarni, U.H. Naik, J.D. Yadhav, S.P. Thorat, A.A. Basade, H.V. Patil, H.T. Dinde**,
A Hand Book of Computational Mathematics Laboratory, Shivaji University
Mathematics Society,2005

4. **Shanti Narayan, P.K. Mittal** :Integral Calculus , S. Chand and comp. New Delhi

5.**B.P. Demidovich& I. A. Maron** Computational Mathematics,translated by George Yankosky
Mir Publishers, Moscow

5. **J.B.Fraleigh**, A first couse in abstract Algebra ,Narosa Publishing House New Delhi, Tenth
Reprint 2003

7. **A.R. Vasishtha** ,Modern Algebra ,Krishna Prakashan ,Meerut 1994

8.**Seymour hipschutz,Marehipson**Schaum'sOutline ,Discrete Mathematics,(3rd Edition), Tata
MaGraw-Hill Publishing Company Ltd., New Delhi.